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A New Type of Topology Via Vague Grill

Hemalatha G¹, Nirmala Irudayam F²

P.G. Student, Department of Mathematics, Nirmala College for Women, Red Fields, Coimbatore, India¹

Assistant Professor, Department of Mathematics, Nirmala College for Women, Red Fields, Coimbatore, India²

ABSTRACT: In this paper, we introduce the concept of vague sets in grill topology. We define a kind of topology obtained as an associated structure on a topological space X induced by a vague grill on X. Such a topology is studied in certain detail as to some of its basic properties.

KEYWORDS: Vague set, Vague topology, Grill topology, Vague Grill topology, Kurssatowski's closure axioms.

I. INTRODUCTION

The idea of grill on a topological space was first introduced by Choquet [4] in 1947. It is observed from literature that the concept of grills is a powerful supporting tool, like nets and filters, in dealing with many-a-topological concept quite effectively. For instance proximity spaces, closure spaces and the theory of compactifications and similar other extension problems are seen to have been tackled excellently by sheer use of grills [2, 3, 8]. Here in this article, we shall define and study a typical topology associated rather naturally with the existing topology and a grill on a given topological space. we introduce a kind of operator which eventually gives rise to another operator which satisfies Kuratowski's closure axioms, thereby inducing a new topology. We study this induced topology to some extent along with some of its basic properties.

The concept of fuzzy sets and its operations are introduced by Zadeh in 1965[9]. Paving path to the new branch, Fuzzy sets, in mathematics Mr.Zadeh along with his co-researchers had set a new trend mark. The concept of Vague set was introduced in 1993 by Gau and Burherer. The basic concept of Vague Set theory and its extensions defined in [5]. The Vague sets approach to artificial intelligence includes Decision Making, Machine Learning and Logical Programming etc.The theory of Vague Topology was introduced by Mariapresenti.L and Arockia Rani.I [6].

II. PRELIMINARIES:

Definition 2.1[7]:

A collection G of a nonempty subsets of a space X is called Grill on X if,

- (i) $A \in G$ and $A \subseteq B \Rightarrow B \in G$
- (ii) $A, B \subseteq X$ and $A \cup B \in G \Rightarrow A \in G$ or $B \in G$.

For any point X of a topological space (X, τ) , $\tau(X)$ denotes the collection of all open neighborhoods of X.

Definition2.2[1]:

A vague set A in the universe of discourse X is characterised by two membership functions given by, a true membership function $t_A : X \rightarrow [0,1]$ and a false membership function $f_A : X \rightarrow [0,1]$. The grade of membership of x in the vague set A is bounded by a sub interval $[t_A(x), 1 - f_A(x)]$ of [0,1]. This indicates that ,if the actual grade of membership $\mu(x)$, then $t_A(x) \leq \mu(x) \leq 1 - f_A(x)$. The vague set A is written as $A = \{x, [t_A(x), 1 - f_A(x)]/x \in X\}$, where the interval $[t_A(x), 1 - f_A(x)]$ is called the vague value of x in A and is denoted by $v_A(x)$

Definition2.3[6][1]:

A vague topology on X is a family τ of vague sets on X satisfying the following conditions:

- (i) $0, 1 \in \tau$
- (ii) $A_1 \cap A_2$ for any $A_1, A_2 \in \tau$.
- (iii) $\cup A_i \in \tau$, for any arbitrary family $\{A_i/, A_i \in \tau, i \in \tau\}$

The pair (X, τ) , is called a vague topological space. The elements of τ are called vague open set.

Definition 2.8[1] :

Let A and B be the vague sets of the form $A = \{x, [t_A(x), 1 - f_A(x)]/x \in X\}$ and $B = \{x, [t_B(x), 1 - f_B(x)]/x \in X\}$ then,

1. $A \subseteq B$ iff $t_A(x) \leq t_B(x)$ and $1 - f_A(x) \leq 1 - f_B(x) \forall x \in X$
2. $A^c = \{x, [f_A(x), 1 - t_A(x)]/x \in X\}$
3. $A \cap B = \{x, [t_A(x) \wedge t_B(x), 1 - f_A(x) \wedge 1 - f_B(x)]/x \in X\}$
4. $A \cup B = \{x, [t_A(x) \vee t_B(x), 1 - f_A(x) \vee 1 - f_B(x)]/x \in X\}$

III. TOPOLOGY INDUCED BY A VAGUE GRILL

Definition 3.1:

A collection G of nonempty subsets of space X, then G is said to be a vague grill on X, if

- i) $\{x, [t_A(x), 1 - f_A(x)]/x \in X\} \in Gand$

$$\{x, [t_A(x) \leq t_B(x), 1 - f_A(x) \leq 1 - f_B(x)]/x \in X\} \Rightarrow \{x, [t_B(x), 1 - f_B(x)]/x \in X\} \in G$$

- ii) $\{x, [t_A(x), 1 - f_A(x)]/x \in X\}, \{x, [t_B(x), 1 - f_B(x)]/x \in X\} \subseteq X$ and $\{x, [t_A(x) \vee t_B(x), 1 - f_A(x) \vee 1 - f_B(x)]/x \in X\} \in G \Rightarrow \{x, [t_A(x), 1 - f_A(x)]/x \in X\} \in G$ (or)

$$\{x, [t_B(x), 1 - f_B(x)]/x \in X\} \in G$$

For any point x of topological space (X, τ) , $\tau(X)$ denotes the collection of all neighbourhoods of X. is said to be vague grill topology

Definition 3.2:

Let (X, τ) be a topological space and G be a vague grill on X. We define a mapping : $\mathcal{P}(X) \rightarrow \mathcal{P}(X)$, denoted by $\phi_G(A, \tau)$, (for $A, \in \mathcal{P}(X)$) or $\phi_G(A)$, or simply by $\phi(A)$, (when it is known which topology and grill on X we are talking about), called the operator associated with the vague grill G and the topology τ , and is defined by

$$\phi_G(A) = \{x \in X : A \cap U \in G, \forall U \in \tau(x)\}.$$

$$\ni A = \{x, [t_A(x), 1 - f_A(x)]/x \in X\}$$

$$\phi_G(\{x, [t_A(x), 1 - f_A(x)]/x \in X\}) = \{x \in X : \{x, [t_A(x), 1 - f_A(x)]/x \in X\} \cap U \in G, \forall U \in \tau(x)\}.$$

Remark 3.3:

Let (X, τ) be a topological space.

- (i) If G is any vague grill on X, then ϕ_G is an increasing function in the sense that $\{x, [t_A(x) \leq t_B(x), 1 - f_A(x) \leq 1 - f_B(x)]/x \in X\} (\subseteq X)$ implies

$$\phi_G(\{x, [t_A(x) \leq t_B(x), 1 - f_A(x) \leq 1 - f_B(x)]/x \in X\}),$$

and if G_1 and G_2 are two vague grills on X with $G_1 \subseteq G_2$,

$$\text{then } \{x, [\phi_{G_1}(t_A(x)) \leq \phi_{G_2}(t_A(x)), \phi_{G_1}(1 - f_A(x)) \leq \phi_{G_2}(1 - f_A(x))]/x \in X\},$$

for all $\{x, [t_A(x), 1 - f_A(x)]/x \in X\} \subseteq X$

- (ii) For any vague grill G on X and any $\{x, [t_A(x), 1 - f_A(x)]/x \in X\} \subseteq X$, if $\{x, [t_A(x), 1 - f_A(x)]/x \in X\} \notin G$ then $\phi_G(\{x, [t_A(x), 1 - f_A(x)]/x \in X\}) = \phi$.

Proposition 3.4:

Let (X, τ) be a topological space and G a vague grill on X. Then for all $\{x, [t_A(x), 1 - f_A(x)]/x \in X\}, \{x, [t_B(x), 1 - f_B(x)]/x \in X\} \subseteq X$,

$$(i) \quad \phi(\{x, [t_A(x) \vee t_B(x), 1 - f_A(x) \vee 1 - f_B(x)]/x \in X\}) = \phi(\{x, [t_A(x), 1 - f_A(x)]/x \in X\}) \cup \phi(\{x, [t_B(x), 1 - f_B(x)]/x \in X\}),$$

$$(ii) \quad \phi(\phi(\{x, [t_A(x), 1 - f_A(x)]/x \in X\})) \subseteq \phi(\{x, [t_A(x), 1 - f_A(x)]/x \in X\}) \\ = \text{cl} \phi(\{x, [t_A(x), 1 - f_A(x)]/x \in X\}) \subseteq \text{cl}\{x, [t_A(x), 1 - f_A(x)]/x \in X\}$$

Proof.

(i) In view of Remark 3.3(i)

it suffices to show that $\phi(\{x, [t_A(x) \vee t_B(x), 1 - f_A(x) \vee 1 - f_B(x)]/x \in X\}) \subseteq \phi(\{x, [t_A(x), 1 - f_A(x)]/x \in X\}) \cup \phi(\{x, [t_B(x), 1 - f_B(x)]/x \in X\})$.

Suppose $x \notin \phi(\{x, [t_A(x), 1 - f_A(x)]/x \in X\}) \cup \phi(\{x, [t_B(x), 1 - f_B(x)]/x \in X\})$. Then there are

$U_1, U_2 \in \tau(x)$ such that $\{x, [t_A(x), 1 - f_A(x)]/x \in X\} \cap U_1, \{x, [t_B(x), 1 - f_B(x)]/x \in X\} \cap U_2 \notin G$ and hence

$(\{x, [t_A(x), 1 - f_A(x)]/x \in X\} \cap U_1) \cup (\{x, [t_B(x), 1 - f_B(x)]/x \in X\} \cap U_2) \notin G$.

$\{x, [t_A(x) \cap U_1 \vee t_B(x) \cap U_2, 1 - f_A(x) \cap U_1 \vee 1 - f_B(x) \cap U_2]/x \in X\} \notin G$

Now $U_1 \cap U_2 \in \tau(x)$ and $(\{x, [t_A(x) \vee t_B(x), 1 - f_A(x) \vee 1 - f_B(x)]/x \in X\}) \cap (U_1 \cap U_2) \subseteq (\{x, [t_A(x), 1 - f_A(x)]/x \in X\} \cap U_1) \cup (\{x, [t_B(x), 1 - f_B(x)]/x \in X\} \cap U_2) \notin G$,

proving that $x \notin \phi(\{x, [t_A(x) \vee t_B(x), 1 - f_A(x) \vee 1 - f_B(x)]/x \in X\})$.

(ii) $x \notin \text{cl}\{x, [t_A(x), 1 - f_A(x)]/x \in X\} \Rightarrow \exists U \in \tau(x)$ such that $U \cap \{x, [t_A(x), 1 - f_A(x)]/x \in X\} = \emptyset \notin G$

$\Rightarrow x \notin \phi(\{x, [t_A(x), 1 - f_A(x)]/x \in X\})$. Thus $\phi(\{x, [t_A(x), 1 - f_A(x)]/x \in X\}) \subseteq \text{cl}\{x, [t_A(x), 1 - f_A(x)]/x \in X\}$.

Now we shall show that $\text{cl}(\phi(\{x, [t_A(x), 1 - f_A(x)]/x \in X\})) \subseteq \phi(\{x, [t_A(x), 1 - f_A(x)]/x \in X\})$.

Indeed, $x \in \text{cl}(\phi(\{x, [t_A(x), 1 - f_A(x)]/x \in X\}))$ and $U \in \tau(x) \Rightarrow U \cap \phi(\{x, [t_A(x), 1 - f_A(x)]/x \in X\}) \neq \emptyset$.

Let $y \in U \cap \phi(\{x, [t_A(x), 1 - f_A(x)]/x \in X\})$, i.e., $y \in U$ and $y \in \phi(\{x, [t_A(x), 1 - f_A(x)]/x \in X\})$.

Then $U \cap \{x, [t_A(x), 1 - f_A(x)]/x \in X\} \in G$ and so $x \in \phi(\{x, [t_A(x), 1 - f_A(x)]/x \in X\})$. Thus $\text{cl}(\phi(\{x, [t_A(x), 1 - f_A(x)]/x \in X\})) = \phi(\{x, [t_A(x), 1 - f_A(x)]/x \in X\})$.

Now, $\phi(\phi(\{x, [t_A(x), 1 - f_A(x)]/x \in X\})) \subseteq \text{cl}(\phi(\{x, [t_A(x), 1 - f_A(x)]/x \in X\})) = \phi(\{x, [t_A(x), 1 - f_A(x)]/x \in X\}) \subseteq \text{cl}\{x, [t_A(x), 1 - f_A(x)]/x \in X\}$.

Let G be a vague grill on a space X . We define a map $\psi : \mathcal{P}(X) \rightarrow \mathcal{P}(X)$ by $\psi(\{x, [t_A(x), 1 - f_A(x)]/x \in X\}) = \{x, [t_A(x), 1 - f_A(x)]/x \in X\} \cup \phi(\{x, [t_A(x), 1 - f_A(x)]/x \in X\})$, for all $\{x, [t_A(x), 1 - f_A(x)]/x \in X\} \in \mathcal{P}(X)$.

Then we have:

Theorem 3.5:

The above map ψ satisfies Kuratowski's closure axioms.

Proof.

By Remark 3.3(ii),

we have $\psi(\emptyset) = \emptyset$, and obviously $\{x, [t_A(x), 1 - f_A(x)]/x \in X\} \subseteq \psi(\{x, [t_A(x), 1 - f_A(x)]/x \in X\})$, $\forall \{x, [t_A(x), 1 - f_A(x)]/x \in X\} \subseteq X$.

Now, $\psi(\{x, [t_A(x) \vee t_B(x), 1 - f_A(x) \vee 1 - f_B(x)]/x \in X\}) = \{x, [t_A(x) \vee t_B(x), 1 - f_A(x) \vee 1 - f_B(x)]/x \in X\} \cup \phi(\{x, [t_A(x) \vee t_B(x), 1 - f_A(x) \vee 1 - f_B(x)]/x \in X\}) = \{x, [t_A(x), 1 - f_A(x)]/x \in X\} \cup \{x, [t_B(x), 1 - f_B(x)]/x \in X\} \cup \phi(\{x, [t_A(x), 1 - f_A(x)]/x \in X\}) \cup \phi(\{x, [t_B(x), 1 - f_B(x)]/x \in X\})$ (by Proposition 3.4(i)) = $\psi(\{x, [t_A(x), 1 - f_A(x)]/x \in X\}) \cup \psi(\{x, [t_B(x), 1 - f_B(x)]/x \in X\})$.

Again for any $\{x, [t_A(x), 1 - f_A(x)]/x \in X\} \subseteq X$, $\psi(\psi(\{x, [t_A(x), 1 - f_A(x)]/x \in X\}))$

= $\psi(\{x, [t_A(x), 1 - f_A(x)]/x \in X\} \cup \phi(\{x, [t_A(x), 1 - f_A(x)]/x \in X\}))$

= $\{x, [t_A(x), 1 - f_A(x)]/x \in X\} \cup \phi(\{x, [t_A(x), 1 - f_A(x)]/x \in X\}) \cup \phi(\{x, [t_A(x), 1 - f_A(x)]/x \in X\} \cup \phi(\{x, [t_A(x), 1 - f_A(x)]/x \in X\}))$

= $\{x, [t_A(x), 1 - f_A(x)]/x \in X\} \cup \phi(\{x, [t_A(x), 1 - f_A(x)]/x \in X\}) \cup \phi(\phi(\{x, [t_A(x), 1 - f_A(x)]/x \in X\}))$ (by Proposition 3.4(i))

= $\{x, [t_A(x), 1 - f_A(x)]/x \in X\} \cup \phi(\{x, [t_A(x), 1 - f_A(x)]/x \in X\})$ (by Proposition 3.4(ii))

= $\psi(\{x, [t_A(x), 1 - f_A(x)]/x \in X\})$.

We thus arrive at the following definition.

Definition 3.6:

Corresponding to a vague grill G on a topological space (X, τ) , there exists a unique topology τ_G (say) on X given by $\tau_G = \{U \subset X : \psi(X \setminus U) = X \setminus U\}$, where for any $\{x, [t_A(x), 1 - f_A(x)]/x \in X\} \subseteq X$, $\psi(\{x, [t_A(x), 1 - f_A(x)]/x \in X\}) = \{x, [t_A(x), 1 - f_A(x)]/x \in X\} \cup \phi(\{x, [t_A(x), 1 - f_A(x)]/x \in X\}) = \tau_G\text{-cl} \{x, [t_A(x), 1 - f_A(x)]/x \in X\}$.

The following theorem shows that for a given vague grill G on a space (X, τ) , the above defined topology τ_G includes the complements of the non-members of G and the complements of $\phi(\{x, [t_A(x), 1 - f_A(x)]/x \in X\})$ (for any $\{x, [t_A(x), 1 - f_A(x)]/x \in X\} \subseteq X$).

Theorem 3.7:

- (a) If G_1 and G_2 are two vague grills on a space X with $G_1 \subseteq G_2$, then $\tau_{G_2} \subseteq \tau_{G_1}$.
- (b) If G is a vague grill on a space X and $\{x, [t_B(x), 1 - f_B(x)]/x \in X\} \notin G$, then $\{x, [t_B(x), 1 - f_B(x)]/x \in X\}$ is closed in (X, τ_G) .
- (c) For any subset $\{x, [t_A(x), 1 - f_A(x)]/x \in X\}$ of a space X and any vague grill G on X , $\phi(\{x, [t_A(x), 1 - f_A(x)]/x \in X\})$ is τ_G -closed.

Proof.

(a) $U \in \tau_{G_2} \Rightarrow \tau_{G_2}\text{-cl}(X \setminus U) = \psi(X \setminus U) \Rightarrow X \setminus U = (X \setminus U) \cup \phi_{G_2}(X \setminus U) \Rightarrow \phi_{G_2}(X \setminus U) \subseteq X \setminus U \Rightarrow \phi_{G_1}(X \setminus U) \subseteq (X \setminus U)$ (by Remark 3.3(i)) $\Rightarrow X \setminus U = \tau_{G_1}\text{-cl}(X \setminus U) \Rightarrow U \in \tau_{G_1}$.

(b) By Remark 3.3(ii), $\{x, [t_B(x), 1 - f_B(x)]/x \in X\} \notin G \Rightarrow \phi(\{x, [t_B(x), 1 - f_B(x)]/x \in X\}) = \phi$, and then $\tau_G\text{-cl} \{x, [t_B(x), 1 - f_B(x)]/x \in X\} = \psi(\{x, [t_B(x), 1 - f_B(x)]/x \in X\}) = \{x, [t_B(x), 1 - f_B(x)]/x \in X\} \cup \phi(\{x, [t_B(x), 1 - f_B(x)]/x \in X\}) = \{x, [t_B(x), 1 - f_B(x)]/x \in X\}$ proving $\{x, [t_B(x), 1 - f_B(x)]/x \in X\}$ to be τ_G -closed.

(c) We have, $\psi(\phi(\{x, [t_A(x), 1 - f_A(x)]/x \in X\})) = \phi(\{x, [t_A(x), 1 - f_A(x)]/x \in X\}) \cup \phi(\phi(\{x, [t_A(x), 1 - f_A(x)]/x \in X\})) = \phi(\{x, [t_A(x), 1 - f_A(x)]/x \in X\})$, (by Proposition 3.4(ii)) $\Rightarrow \phi(\{x, [t_A(x), 1 - f_A(x)]/x \in X\})$ is τ_G -closed.

We know find a suitable open base for the topology τ_G

Theorem 3.8.

Let (X, τ) be a topological space and G be a vague grill on X . Then $\mathcal{B}(G, \tau) = \{V \setminus \{x, [t_A(x), 1 - f_A(x)]/x \in X\} : V \in \tau \text{ and } \{x, [t_A(x), 1 - f_A(x)]/x \in X\} \notin G\}$ is an open base for τ_G .

Proof.

Let $U \in \tau_G$ and $x \in U$. Then $(X \setminus U)$ is τ_G -closed so that $\psi(X \setminus U) = X \setminus U$, and hence $\Phi(X \setminus U) \subseteq X \setminus U$. Then $x \notin \phi(X \setminus U)$ and so there exists $V \in \tau(x)$ such that $(X \setminus U) \cap V \notin G$.

Let $\{x, [t_A(x), 1 - f_A(x)]/x \in X\} = (X \setminus U) \cap V$, then $x \notin \{x, [t_A(x), 1 - f_A(x)]/x \in X\}$ and $\{x, [t_A(x), 1 - f_A(x)]/x \in X\} \notin G$. Thus $x \in V \setminus \{x, [t_A(x), 1 - f_A(x)]/x \in X\} = V \setminus [(X \setminus U) \cap V] = V \setminus (X \setminus U) \subseteq U$, where $V \setminus \{x, [t_A(x), 1 - f_A(x)]/x \in X\} \in \mathcal{B}(G, \tau)$.

It now suffices to observe that $\{x, [t_B(x), 1 - f_B(x)]/x \in X\} (G, \tau)$ is closed under finite intersections.

Let $V_1 \setminus \{x, [t_A(x), 1 - f_A(x)]/x \in X\}, V_2 \setminus \{x, [t_B(x), 1 - f_B(x)]/x \in X\} \in \mathcal{B}(G, \tau)$, i.e., $V_1, V_2 \in \tau$ and $\{x, [t_A(x), 1 - f_A(x)]/x \in X\}, \{x, [t_B(x), 1 - f_B(x)]/x \in X\} \notin G$. Then $V_1 \cap V_2 \in \tau$ and $\{x, [t_A(x), 1 - f_A(x)]/x \in X\} \cup \{x, [t_B(x), 1 - f_B(x)]/x \in X\} \notin G$.

Now, $(V_1 \setminus \{x, [t_A(x), 1 - f_A(x)]/x \in X\}) \cap (V_2 \setminus \{x, [t_B(x), 1 - f_B(x)]/x \in X\}) = (V_1 \cap V_2) \setminus (\{x, [t_A(x), 1 - f_A(x)]/x \in X\} \cup \{x, [t_B(x), 1 - f_B(x)]/x \in X\}) \in \mathcal{B}(G, \tau)$, proving ultimately that $\mathcal{B}(G, \tau)$ is an open base for τ_G .

Corollary 3.9

For any vague grill G on a topological space (X, τ) , $\tau \subseteq \mathcal{B}(G, \tau) \subseteq \tau_G$.

Example 3.10.

Let (X, τ) be a topological space. If $G = P(X) \setminus \{\phi\}$, then $\tau_G = \tau$. In fact, for any τ_G -basic open set $V = U \setminus \{x, [t_A(x), 1 - f_A(x)]/x \in X\}$ with $U \in \tau$ and $\{x, [t_A(x), 1 - f_A(x)]/x \in X\} \notin G$, we have $\{x, [t_A(x), 1 - f_A(x)]/x \in X\} = \phi$, so that $V = U \in \tau$. Hence in view of Corollary 3.9, we have in this case $\tau = \mathcal{B}(G, \tau) = \tau_G$.

In what follows in the section, we shall deal with certain properties concerning the topology τ_G and the operator ϕ .

Theorem 3.11

If G is a grill on a space (X, τ) with $\tau \setminus \{\emptyset\} \subseteq G$, then for all $U \in \tau, U \subseteq \Phi(U)$.

Proof.

In case $U = \emptyset$, we obviously have $\Phi(U) = \emptyset = U$. Now note that if $\tau \setminus \{\emptyset\} \subseteq G$, then $\Phi(X) = X$.

In fact, $x \notin \Phi(X) \Rightarrow \exists V \in \tau(x)$ such that $V \cap X \notin G$

$\Rightarrow V \notin G$, a contradiction.

Now, by using Theorem 3.10 we have for any $U \in \tau \setminus \{\emptyset\}, U \cap \Phi(X) = U \cap \Phi(U \cap X)$, and hence by above, $U = U \cap X = U \cap \Phi(U)$. Thus $\Phi(U) \supseteq U$.

IV. TOPOLOGY SUITABLE FOR A VAGUE GRILL

In this section we shall consider vague grills satisfying a certain condition and this will make the vague grills more suitable and compatible vis-a-vis the topology of the space. We intend to do some investigations in respect of the topology τ_G , along with certain applications, under the assumption of such a suitability condition imposed on the concerned vague grills.

Definition 4.1.

Let G be a vague grill on a topological space (X, τ) . Then τ is said to be suitable for the vague grill G if for all $\{x, [t_A(x), 1 - f_A(x)]/x \in X\} \subseteq X, \{x, [t_A(x), 1 - f_A(x)]/x \in X\} \setminus \phi(\{x, [t_A(x), 1 - f_A(x)]/x \in X\}) \notin G$.

We now give some equivalent descriptions of the above definition.

Theorem 4.2.

For a vague grill G on a space (X, τ) , the following are equivalent:

- (a) τ is suitable for the grill G
- (b) For any τ_G -closed subset $\{x, [t_A(x), 1 - f_A(x)]/x \in X\}$ of $X, \{x, [t_A(x), 1 - f_A(x)]/x \in X\} \setminus \phi(\{x, [t_A(x), 1 - f_A(x)]/x \in X\}) \notin G$.
- (c) Whenever for any $\{x, [t_A(x), 1 - f_A(x)]/x \in X\} \subseteq X$ and each $x \in \{x, [t_A(x), 1 - f_A(x)]/x \in X\}$ there corresponds some $U_x \in \tau(x)$ with $U_x \cap A \notin G$, it follows that $\{x, [t_A(x), 1 - f_A(x)]/x \in X\} \notin G$.
- (d) $\{x, [t_A(x), 1 - f_A(x)]/x \in X\} \subseteq X$ and $\{x, [t_A(x), 1 - f_A(x)]/x \in X\} \cap \phi(\{x, [t_A(x), 1 - f_A(x)]/x \in X\}) = \emptyset \Rightarrow \{x, [t_A(x), 1 - f_A(x)]/x \in X\} \notin G$.

Proof.

(a) \Rightarrow (b): It is trivial.

(b) \Rightarrow (c):

Let $\{x, [t_A(x), 1 - f_A(x)]/x \in X\} \subseteq X$ and assume that for every $x \in \{x, [t_A(x), 1 - f_A(x)]/x \in X\}$ there exists $U \in \tau(x)$ such that $U \cap \{x, [t_A(x), 1 - f_A(x)]/x \in X\} \notin G$. Then $x \notin \phi(\{x, [t_A(x), 1 - f_A(x)]/x \in X\})$ so that $\{x, [t_A(x), 1 - f_A(x)]/x \in X\} \cap \phi(\{x, [t_A(x), 1 - f_A(x)]/x \in X\}) = \emptyset$. Now as $\{x, [t_A(x), 1 - f_A(x)]/x \in X\} \cup \phi(\{x, [t_A(x), 1 - f_A(x)]/x \in X\})$ is τ_G -closed, by (b) we have $(\{x, [t_A(x), 1 - f_A(x)]/x \in X\} \cup \phi(\{x, [t_A(x), 1 - f_A(x)]/x \in X\})) \setminus \phi(\{x, [t_A(x), 1 - f_A(x)]/x \in X\}) \notin G$ i.e., $(\{x, [t_A(x), 1 - f_A(x)]/x \in X\} \cup \phi(\{x, [t_A(x), 1 - f_A(x)]/x \in X\})) \setminus (\phi(\{x, [t_A(x), 1 - f_A(x)]/x \in X\}) \cup \phi(\phi(\{x, [t_A(x), 1 - f_A(x)]/x \in X\}))) \notin G$ (by Proposition 2.3(i)) i.e., $(\{x, [t_A(x), 1 - f_A(x)]/x \in X\} \cup \phi(\{x, [t_A(x), 1 - f_A(x)]/x \in X\})) \setminus \phi(\{x, [t_A(x), 1 - f_A(x)]/x \in X\}) \notin G$ (by Proposition 2.3(ii)) i.e., $\{x, [t_A(x), 1 - f_A(x)]/x \in X\} \notin G$ (as $A \cap \phi(A) = \emptyset$).

(c) \Rightarrow (d):

If $\{x, [t_A(x), 1 - f_A(x)]/x \in X\} \subseteq X$ and $\{x, [t_A(x), 1 - f_A(x)]/x \in X\} \cap \phi(\{x, [t_A(x), 1 - f_A(x)]/x \in X\}) = \emptyset$, then $\{x, [t_A(x), 1 - f_A(x)]/x \in X\} \subseteq X \setminus \phi(\{x, [t_A(x), 1 - f_A(x)]/x \in X\})$.

Let $x \in \{x, [t_A(x), 1 - f_A(x)]/x \in X\}$. Then $x \notin \phi(\{x, [t_A(x), 1 - f_A(x)]/x \in X\}) \Rightarrow \exists U \in \tau(x)$ such that $U \cap \{x, [t_A(x), 1 - f_A(x)]/x \in X\} \notin G$. Then by (c), $\{x, [t_A(x), 1 - f_A(x)]/x \in X\} \notin G$.

(d) \Rightarrow (a):

Let $\{x, [t_A(x), 1 - f_A(x)]/x \in X\} \subseteq X$. We first claim that $(A\{x, [t_A(x), 1 - f_A(x)]/x \in X\} \setminus \phi(\{x, [t_A(x), 1 - f_A(x)]/x \in X\})) \cap \phi(\{x, [t_A(x), 1 - f_A(x)]/x \in X\} \setminus \phi(\{x, [t_A(x), 1 - f_A(x)]/x \in X\})) = \phi$.

In fact,

$x \in (\{x, [t_A(x), 1 - f_A(x)]/x \in X\} \setminus \phi(\{x, [t_A(x), 1 - f_A(x)]/x \in X\})) \cap \phi(\{x, [t_A(x), 1 - f_A(x)]/x \in X\} \setminus \phi(\{x, [t_A(x), 1 - f_A(x)]/x \in X\})) \dots (i)$

$\Rightarrow x \in \{x, [t_A(x), 1 - f_A(x)]/x \in X\} \setminus \phi(\{x, [t_A(x), 1 - f_A(x)]/x \in X\}) \Rightarrow x \in \{x, [t_A(x), 1 - f_A(x)]/x \in X\}$ and $x \notin \phi(\{x, [t_A(x), 1 - f_A(x)]/x \in X\}) \Rightarrow \exists U \in \tau(x)$ such that $U \cap \{x, [t_A(x), 1 - f_A(x)]/x \in X\} \notin G$.

Now, $U \cap (\{x, [t_A(x), 1 - f_A(x)]/x \in X\} \setminus \phi(\{x, [t_A(x), 1 - f_A(x)]/x \in X\})) \subseteq U \cap \{x, [t_A(x), 1 - f_A(x)]/x \in X\} \notin G \Rightarrow x \notin \phi(\{x, [t_A(x), 1 - f_A(x)]/x \in X\} \setminus \phi(\{x, [t_A(x), 1 - f_A(x)]/x \in X\}))$, contradicting (i).

Hence by (d), $\{x, [t_A(x), 1 - f_A(x)]/x \in X\} \setminus \phi(\{x, [t_A(x), 1 - f_A(x)]/x \in X\}) \notin G$.

V. CONCLUSION

In this paper, we have introduced the concept of the vague sets in grill topology. We have defined a kind of topology obtained as an associated structure on a topological space X induced by a vague grill on X , and we also discussed some of its basic properties.

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